

# Characteristics of good and competitive equilibrium

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## 1 Introduction

This paper analyses the effect on a competitive equilibrium of a change in the characteristics of goods. Such a change can be interpreted in different ways. For example, advertising may modify the characteristics (see Luski-Wettstein (1986)). More generally any new information modifies the characteristics (Stigler & Becker (1977)). To simplify the interpretation, we assume that information is exchanged at zero cost. The internet is a good example.

We consider a pure exchange economy. Preferences of the agents depend on the characteristics of goods (Lancaster (1966), Michael and Becker (1973)). These characteristics depend on the information hold by the agent. New freely available information modifies these characteristics, welfare of agents and equilibrium.

We show that more information increases welfare (in fact, a linear combination of the utilities of the agents). But it does not necessarily benefit the agent whose information has increased.

Section 2 presents the model, effects are analysed in Section 3 and an example is studied in Section 4.

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## 2 The model

We assume that the preferences of consumer  $i$ ,  $i = 1, 2, \dots, m$  depend on the characteristics of goods and information modifies these characteristics. There is a transformation of market goods into characteristics. Information is an input in the agents' production function of commodities.

Like Luski and Wettstein (1994), we assume that the effect of information  $I$  is only tied to good 1. We identify the other commodities with characteristics:  $Z_h^i = C_h^i$ ;  $h = 2, \dots, n$ ,  $C_h^i$  is the consumed quantity of good  $h$ . For commodity 1, we have (see Luski and Wettstein (1994), p.311):

$$Z_1^i = g(I^i)C_1 = g^i C_1, \quad g^i > 0.$$

An increase of information increases  $g^i$ . The utility function of agent  $i$  is

$$U^i = U^i(g^i C_1^i, C_2^i, \dots, C_n^i).$$

Agent  $i$ 's endowment is  $w^i = (w_1^i, w_2^i, \dots, w_n^i)$ . Taking good 1 as a numeraire, his budget constraint is

$$C_1^i - w_1^i + \sum_{h=2}^n p_h (C_h^i - w_h^i) = 0 \quad (1)$$

where  $p_h$  is the price of good  $h$ . Assuming that all prices  $p_h$  are positive, that the utility function is strictly concave, differentiable and verifies the Inada conditions, the behaviour of agent  $i$  is characterized by the following first order conditions:

$$g^i U_1^i = \frac{1}{p_h} U_h^i, \quad h = 2, \dots, n \quad (2)$$

where  $U_1^i, \dots, U_n^i$  are the partial derivatives of  $U^i$ .

In equilibrium, the quantities  $(C_1^i, C_2^i, \dots, C_n^i)$ ,  $i = 1, \dots, m$  verify

$$\sum_{i=1}^m C_h^i = \sum_{i=1}^m w_h^i, \quad h = 1, \dots, n.$$

## 3 Effect of an increase of information

Let us first consider the effect of a change in prices  $(p_2, \dots, p_n)$ . Differentiating (1) leads to

$$dC_1^i + \sum_{h=2}^n dp_h (C_h^i - w_h^i) + \sum_{h=2}^n p_h dC_h^i = 0, \quad (3)$$

and the marginal effect of a change in prices on the utility  $U^i$  is

$$d_p U^i = g^i U_1^i dC_1^i + \sum_{h=2}^n U_h^i dC_h^i.$$

Substituting (3) for  $dC_1^i$  into this expression leads to :

$$d_p U^i = g^i U_1^i \left[ \sum_{h=2}^n dp_h (w_h^i - C_h^i) - \sum_{h=2}^n p_h dC_h^i \right] + \sum_{h=2}^n U_h^i dC_h^i.$$

Using conditions (2) we see that

$$d_p U^i = g^i U_1^i \sum_{h=2}^n dp_h (w_h^i - C_h^i). \quad (4)$$

The effect of a change in information combines the direct effect on the utility  $U^i$  and the change induced on equilibrium prices. The effects on equilibrium prices are complex (these effects will be analysed in an example). Assuming that the equilibrium exists, we can show that there is an increase in welfare in the following sense.

**Proposition 1** *If the information of at least one agent increases and if the information of the others agents does not decrease, then necessarily the utility of at least one agent increases. More precisely, a linear combination with positive coefficients of the utilities of the  $m$  agents increases.*

**Proof:** Consider a marginal change in information of agent  $i$ ,  $dg^i \geq 0$ ,  $i = 1, \dots, m$ . The corresponding change in utility of agent  $i$  in equilibrium is

$$dU^i = C_1^i U_1^i dg^i + d_p U^i, \quad (5)$$

where  $d_p U^i$  is the effect of the change of equilibrium prices on utility (4). We also have

$$\sum_{i=1}^m \frac{1}{g^i U_1^i} d_p U^i = \sum_{i=1}^m \sum_{h=2}^n (w_h^i - C_h^i) dp_h = 0 \quad (6)$$

since in equilibrium,  $\sum_{i=1}^m (w_h^i - C_h^i) = 0$ ,  $h = 1, \dots, n$ . For any sequence  $\lambda_i$ ,  $i = 1, \dots, m$ , (5) implies

$$\sum_{i=1}^m \lambda_i dU^i = \sum_{i=1}^m \lambda_i C_1^i U_1^i dg^i + \sum_{i=1}^m \lambda_i d_p U^i. \quad (7)$$

Thus, with  $\lambda_i = 1/(g^i U_1^i) > 0$ , (5) and (6) imply

$$\sum_{i=1}^m \frac{1}{g^i U_1^i} dU^i = \sum_{i=1}^m \frac{C_1^i}{g^i} dg^i > 0$$

if all  $dg^i \geq 0$  and at least one  $dg^i > 0$ . □

## 4 An example

In the following simple example with two agents and two goods, we assume that only agent 1 gathers more information ( $dg^1 > 0, dg^2 = 0$ )<sup>1</sup>. This example exhibits different cases: the utility of both agents increases, or the utility of the agent 1 only increases or it decreases. Consider

$$U^i = \frac{(g^i C_1^i)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \frac{(C_2^i)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \quad i = 1, 2, \quad \sigma > 0 \quad \sigma \neq 1$$

$$(w_1^1, w_2^1) = (b, 1) \quad \text{and} \quad (w_1^2, w_2^2) = (1, b), \quad b > 0.$$

For  $g^1 = g^2 = 1$ , the competitive equilibrium is:

$$p = 1, \quad C_1^1 = C_1^2 = C_1^2 = C_2^2 = \frac{1+b}{2} \equiv \mu.$$

The equilibrium conditions for  $g^1 \neq g^2 = 1$  are

$$C_1^1 - b + p(C_2^1 - 1) = 0, \quad C_1^1 + C_1^2 = 2\mu = C_2^1 + C_2^2$$

$$pg^1(g^1 C_1^1)^{-\frac{1}{\sigma}} = (C_2^1)^{-\frac{1}{\sigma}}, \quad p(C_1^2)^{-\frac{1}{\sigma}} = (C_2^2)^{-\frac{1}{\sigma}}$$

By differentiating these conditions we obtain the effect of a marginal change of  $g^1$  in the neighborhood of the equilibrium ( $g^1 = g^2 = 1, p = 1, C_1^1 = C_1^2 = C_2^1 = C_2^2 = \mu$ ):

$$dp = \frac{1}{2} \left( \frac{1}{\sigma} - 1 \right) dg^1,$$

$$dC_1^1 = \frac{-1}{4} (\mu - 1 + \sigma\mu) \left( \frac{1}{\sigma} - 1 \right) dg^1,$$

$$dC_1^2 = -dC_1^1, \quad dC_2^1 = (1 - \mu)dp - dC_1^1, \quad dC_2^2 = -dC_2^1.$$

Using  $dU^1 = \mu^{-\frac{1}{\sigma}} [dC_1^1 + \mu dg^1 + dC_2^1]$  and  $dU^2 = \mu^{-\frac{1}{\sigma}} [dC_1^2 + dC_2^2]$ , we obtain

$$\begin{cases} dU^1 = \frac{\mu^{-\frac{1}{\sigma}}}{4\sigma} \left[ \mu + \frac{1}{2}(1 - \mu) \left( \frac{1}{\sigma} - 1 \right) \right] dg^1 = \frac{\mu^{-\frac{1}{\sigma}}}{4\sigma} [2\sigma(b + 1) + (\sigma - 1)(b - 1)] dg^1, \\ dU^2 = \frac{\mu^{-\frac{1}{\sigma}}}{4\sigma} (\mu - 1) dp = \frac{\mu^{-\frac{1}{\sigma}}}{4\sigma} [(1 - \sigma)(b - 1)] dg^1. \end{cases}$$

<sup>1</sup> We could assume that there are two types of agents.

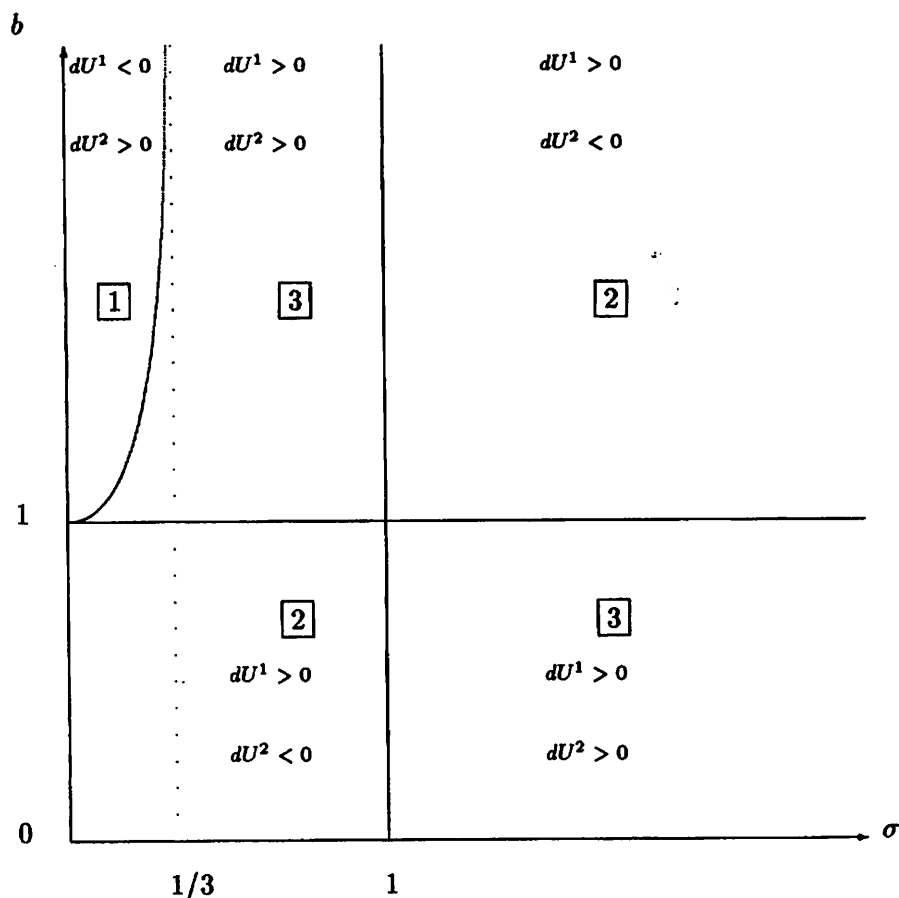


Figure 1

Figure 1 shows the welfare changes in the space of parameters  $\sigma$  and  $b$ , when information increases for agent 1 only.

In region [1] the utility of agent 1 decreases. All utilities increase in regions [3], but the utility of agent 2 who has no new information decreases in regions [2].

The relative price  $p$  of good 2 increases (decreases) when the elasticity of substitution  $\sigma$  is smaller (larger) than 1. For agent 1, there is a direct positive effect of  $dg^1$  on her utility. With an increase in  $p$  (case  $\sigma < 1$ ) the induced effect is negative (positive) when she is a net buyer (seller) of good 2, i.e.  $b > 1$  ( $b < 1$ ). In the case  $\sigma < 1$  and  $b > 1$ , this negative effect may dominate the direct positive effect, when  $\sigma$  is small enough (region [1]).

For agent 2, there is only the indirect effect of the change in price  $p$ , and a similar interpretation applies.

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